Notes from *Applied Combinatorics*

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1 Introduction

This document concerns my interactions with chapter 3 from *Applied Combinatorics* by Fred S. Roberts. Specifically, I've made some quick notes as to the exercises I did or skipped during my process of reading chapter 3. That is, if I completed a number of exercises from a section, I have posted my solutions here; if I skipped a section's exercises, it is because I had already learned the concept the section was covering and felt no further practice was, as of week 1 for this project, necessary. For my previous Graph theory experience, please see *An Introduction to Graph Theory* by Richard J. Trudeau, the dover edition. To examine my solutions, please skip to the appropriate section, and to read about my process, please examine the following paragraph.

My reading process for mathematics textbooks involves the following: reading the book or section of it; highlighting definitions, theorems, corollaries, choice examples, etc.; rewriting the definitions, theorems, etc. in a notebook; and doing as many exercises until I feel I understand the concept (this involves a mixture of simple to complex exercises).

Chapter 3 Section 1: Fundamental Concepts

These exercises were skipped, as Trudeau's book covers these concepts nicely. His notation, however, was different. Furthermore, despite not covering directed graphs, the concepts and notation of graphs translated nicely to digraphs. This section, however, did not have any exercises expanding on what I already know; hence, they were skipped.

Chapter 3 Section 2: Connectedness

For this section, I felt comfortable with the concepts because of my course on Ramsey theory and from Trudeau's book. However, I decided to do a few of the proof exercises so I can become accustomed to Graph theory's proof techniques. Trudeau's book did involve some proofs but the proof techniques he introduced in that book were fairly casual and I prefer being rigorous.

- 3.2.11: If v is reachable from u in a digraph, then there is a simple path from u to v in D.

  Proof. Suppose D is a digraph such that $D = (V, A)$
  Suppose u, v are elements of V such that v is reachable from u
  This means there exists some path from u to v
  Select this path, we know it is of the form $u, u_1, u_2, \cdots, u_t, v$, $0 \leq t$.
  Note, this path may already be simple, or there may be some repeated vertex within the path.
  Now, suppose we start at u and begin traveling along our path.
  Going from u to the next vertex

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- Personal Question: when is it appropriate to use an algorithm in a proof? If the proof is constructing an algorithm, must every step of the algorithm be justified in terms of a proof? Does that make it acceptable?

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Chapter 3 Section 3: Graph Coloring and Its Applications

Chapter 3 Section 4: Chromatic Polynomials

- Personal Question: in Figure 3.52, pg. 115, could a and b be joined in another way? Or do the vertices always join at a?

Chapter 3 Section 5: Trees

- Having had a data structures and an algorithms class, this chapter was review.

Chapter 3 Section 6: Applications of Trees to Searching and Sorting Problems

- Again, having had a data structures and an algorithms class, this chapter was review.

Chapter 3 Section 7: Representing a Graph in the Computer