(1,2)-Step Competition Graph Examples

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1 Introduction

This document is a web of examples constructed from the definitions found in *The (1,2)-step competition number of a graph* by Factor et al.

2 Definitions

Note: before continuing with the definitions, we let D represent an arbitrary digraph, $V(D)$ be its vertex set, and $A(D)$ be its arc set. Furthermore, we let G represent an arbitrary graph, $V(G)$ be its vertex set, and $E(G)$ be its edge set. We let K_n denote a complete graph (digraph) of n vertices; we will specify whether we are using a graph or digraph. These objects are what we will work with unless otherwise specified.

Definition 2.1. Competition graph: the *competition graph of* D, $C(D)$ is the graph with vertex set $V(D)$ such that $\{u, v\}$ is an edge if vertices u and v have a common prey in D.

Definition 2.2. Competition number: the *competition number* $k(G)$ of a graph G is the smallest nonnegative integer k so that G, together with k isolated vertices, is the competition graph of some acyclic digraph.

Definition 2.3. (1,2)-step competition graphs or $C_{1,2}$(D) a graph with a vertex set V(D) in which distinct vertices x and y will make an edge $\{x,y\}$ only when some vertex $(z) \in$ *V(D),* $d_{D-y}(x,y) \leq 2$ and $d_D(y,z) = 1$ or $d_D - x(y,z) \leq 2$ and d_D (x,z) =1.

Definition 2.4. The (1,2)-step competition graph can be generalized as a (i, m)-step competition graph: if for some $z \in V(T) - \{x, y\}$, $d_{T-y}(x, z) \leq i$ and $d_{T-x}(y, z) \leq m$ or $d_{T-x}(y, z) \leq i$ and $d_{T-y}(x, z) \leq m$.

Definition 2.5. Out-Neighborhood of vertex x: the set of all vertices to which x has an arc in D, denoted $N_D^+(x)$

Definition 2.6. In-Neighborhood of a vertex x: the set of all vertices from which x has an arc in D, denoted $N_D^-(x)$

Definition 2.7. Outdegree of a vertex x: the cardinality of the out-neighborhood of x, denoted $\deg_D^+(x)$

Definition 2.8. Indegree of a vertex x: the cardinality of the in-neighborhood of x, denoted $\deg_D^-(x)$

Definition 2.9. (i,m)-step competition number: the (i,m)-*step competition number* of a graph G is the minimum number k such that G with k isolated vertices is the (i,m)-step competition graph of an acyclic digraph.

3 Lemmas

Lemma 3.1. For all positive integers $n \geq 5, k_{(1,2)}(C_n) > 1$

4 Propositions

Proposition 4.1. Let i, m be positive integers. For any graph $4G, there exists a nonnegative integer$ k$such that 4G$ with k isolated vertices in the (i, m)-step competition graph of an acyclic digraph.

Proposition 4.2. Let i, m be positive integers. If a graph G has no isolated vertices, then $k_{(i,m)}(G) \geq 1$.

5 Theorems

Theorem 5.1. Let i, m be positive integers. For $n \geq 2$, we have $k_{(i,m)}(K_n) = 1$.

Theorem 5.2. For all positive integers $n > 1, k_{(1,2)}(P_n)$.

Theorem 5.3. Let $n \geq 3$ be a positive integer. Then $k_{(1,2)}(C_n) = 1$ when $n3, 4$. For $n \geq 5, k_{(1,2)}(C_n) = 2$.

Theorem 5.4. Let n be a positive integer. then $k_{(1,2)}(K_{1,n}) = 1$.