DUAL FILTERING DATA ASSIMILATION OF DYNAMIC SYSTEMS

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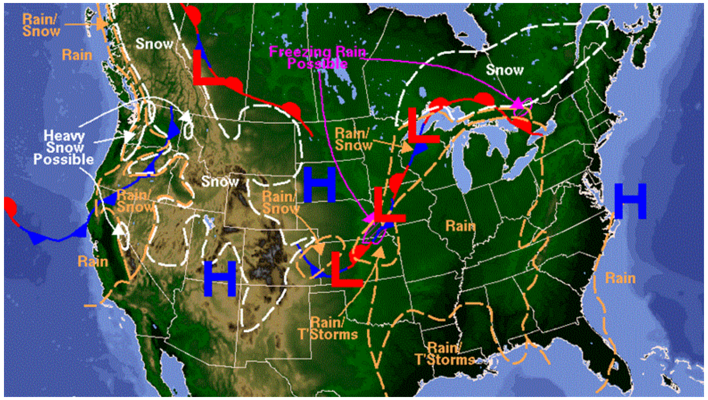
WHAT IS DATA ASSIMILATION?

- **Data Assimilation: model (x^b) and observational (y) analysis ➔ forecasting (x^a) [1]**

- **Applicable to real world: weather [2]**

- **Kalman Filter**



[1] *Data Assimilation Part 1…* Spiller [2] *Ensemble Based Forecasting…* Hamill
Image: Weekly Weather Forecast… Standard Examiner

KALMAN FILTER

- **Gaussian (normal) observations and projections**
- **Derived from manipulation of Baye's Rule:**

$$\mathrm{P}(\mathrm{x}_t \mid \psi_t) \propto P(y_t \mid x_t)\mathrm{P}(\mathrm{x}_t \mid \psi_{t-1})$$

- **Following assimilation equation (1D) [2]:**

$$x^a = x^b + \frac{P^b}{P^b + R}(y - x^b)$$

[2] *Ensemble Based Forecasting…* Hamill

ENSEMBLE KALMAN FILTER

- As Dimensions increase ➔ reliability decreases

- Run the filter with an ensemble of x^b for i members [1]:

$$x_i^a = x_i^b + \hat{K}(y + \eta_i - x_i^b)$$

- Why does this work?

[1] *Data Assimilation Part 1…* Spiller

RESEARCH GOAL

- **Show why higher dimensional Ensemble Kalman Filter works**

- **Experiment with a system:**
  - Parameterization ➔ Particle Filter [4]

- **Increase accuracy ➔ Decrease variance between points**

[4] *Two–Stage Filtering for Joint…* Santitissadeekorn

SYSTEM IN QUESTION LORENZ '96

- **Cyclical system of ordinary differential equations**
- **Represents atmospheric behavior [3]:**

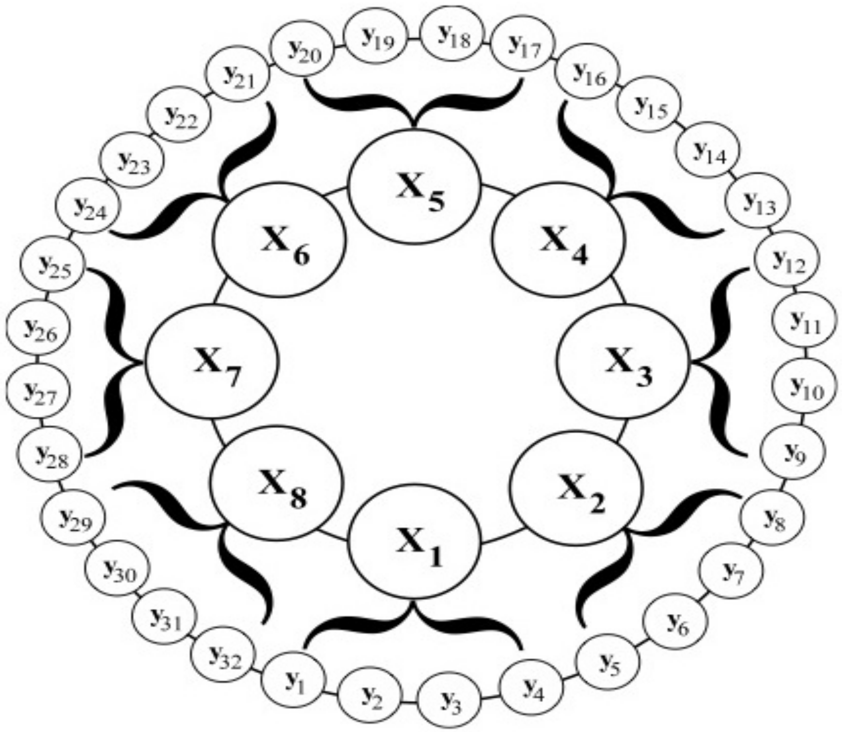
$$\frac{dx_i}{dt} = x_{i-1}(x_{i+1} - x_{i-2}) - x_i + F - \frac{hc}{b} \sum_{j=J(i-1)+1}^{iJ} y_j$$

$$\frac{dy_j}{dt} = -cby_{j+1}(y_{j+2} - y_{j-1}) - cy_j + \frac{hc}{b} x_{floor[(j-1)/J]+1}$$

- **I=1, 2… i (slow variables) J=1, 2… j (fast variables)**

[3] *Aggressive shadowing of a Low…* Lieb-Lappen

LORENZ '96 VISUAL [3]



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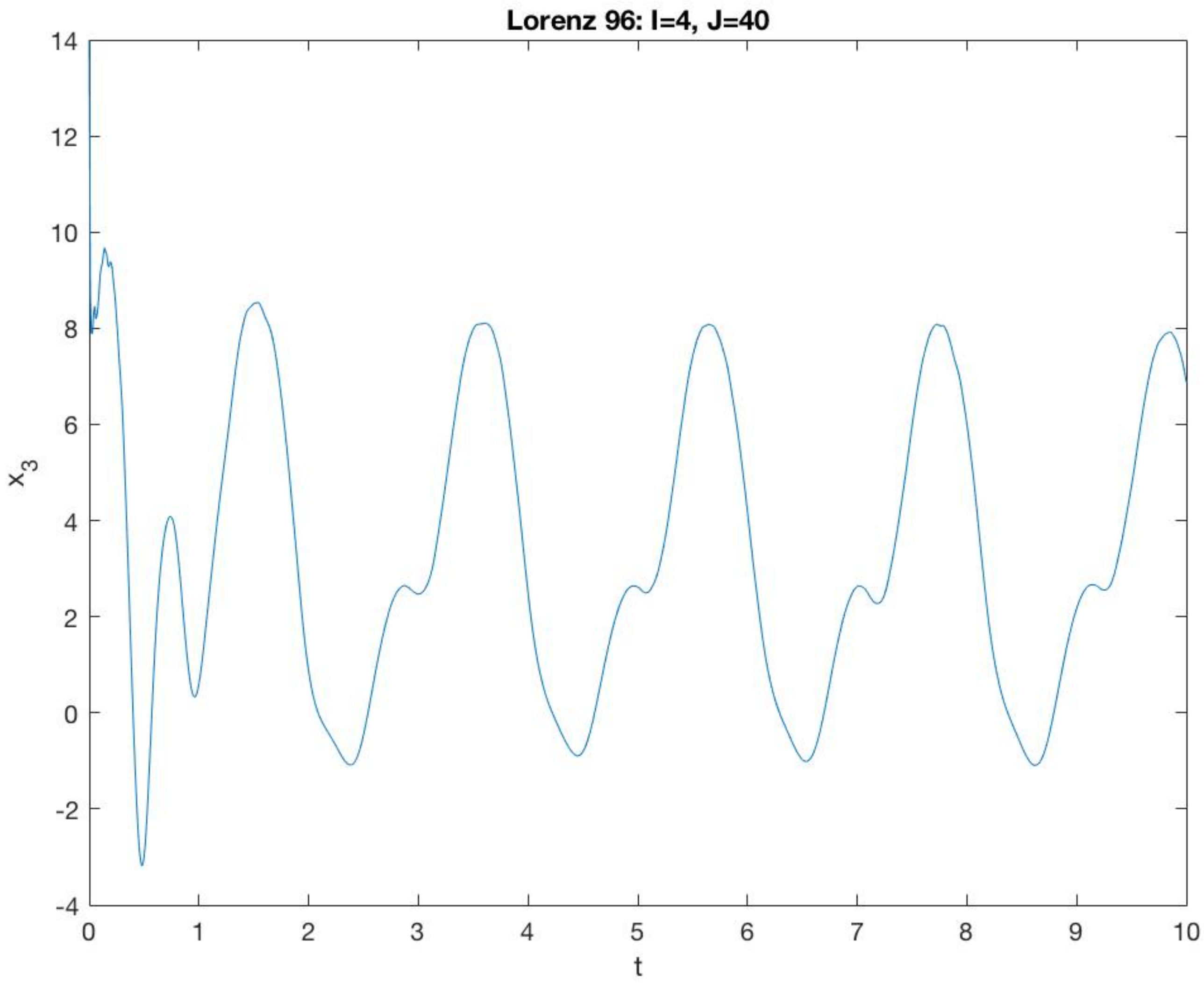
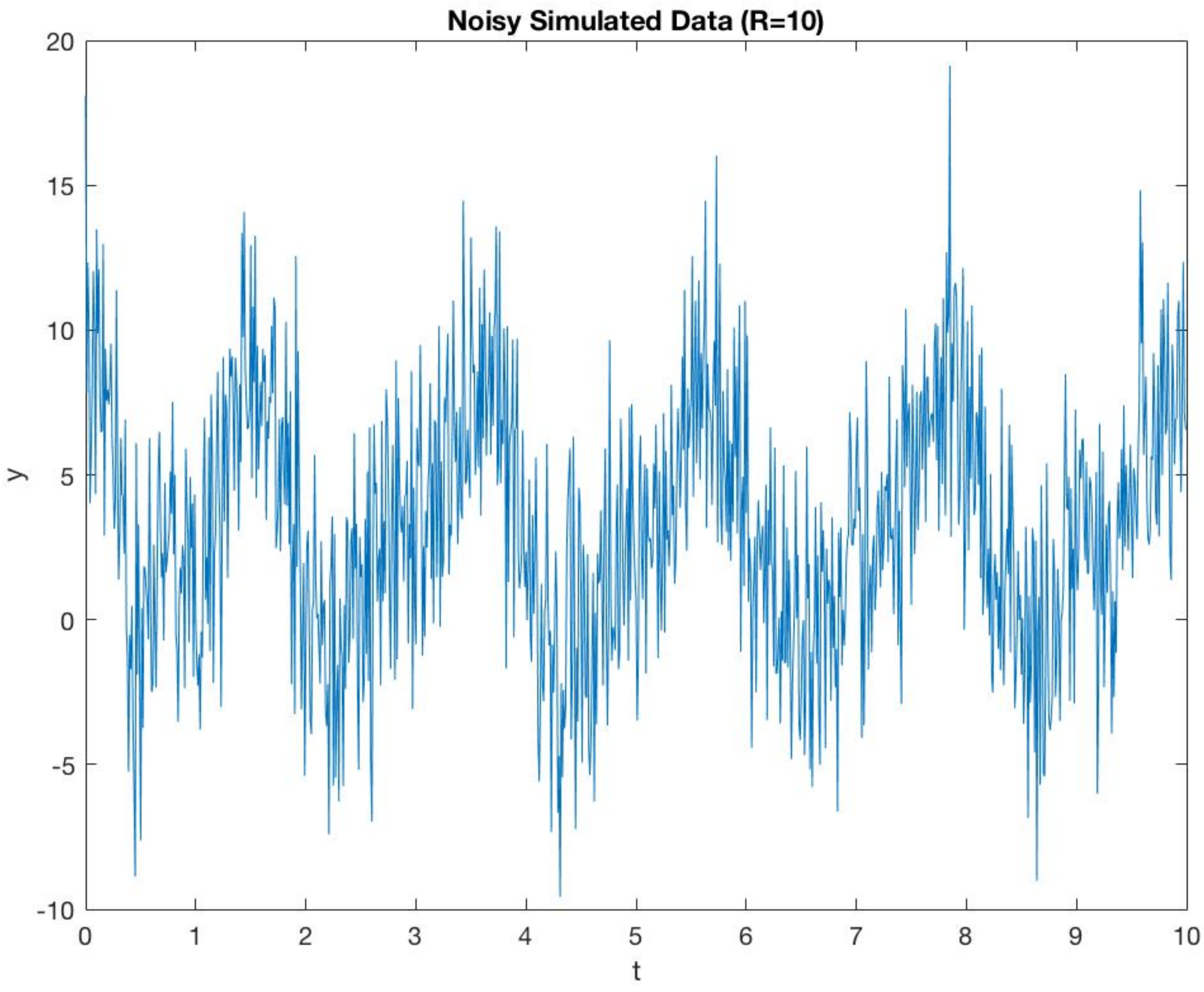
LORENZ '96 VISUAL



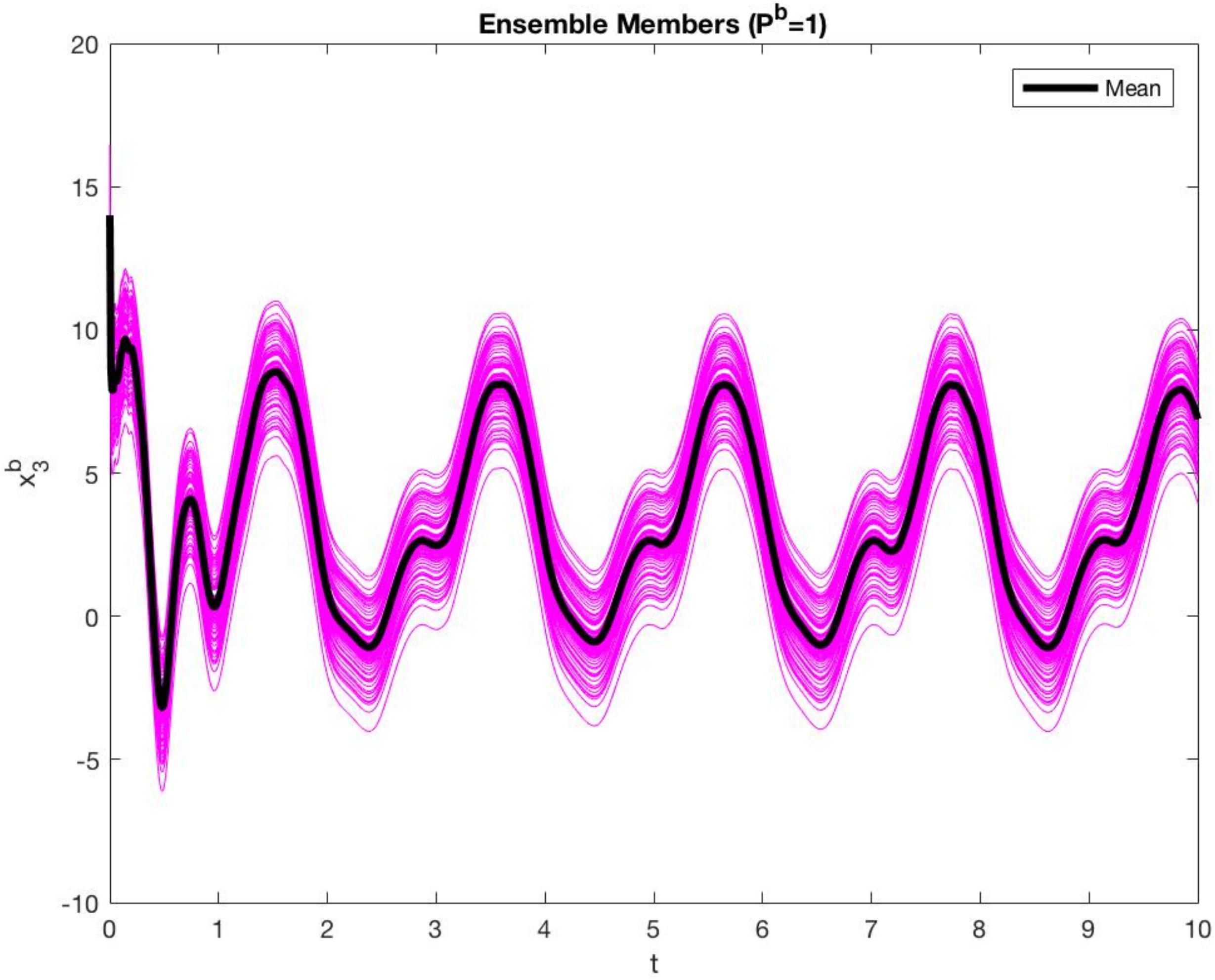
METHODS

- **Simulate observational data**



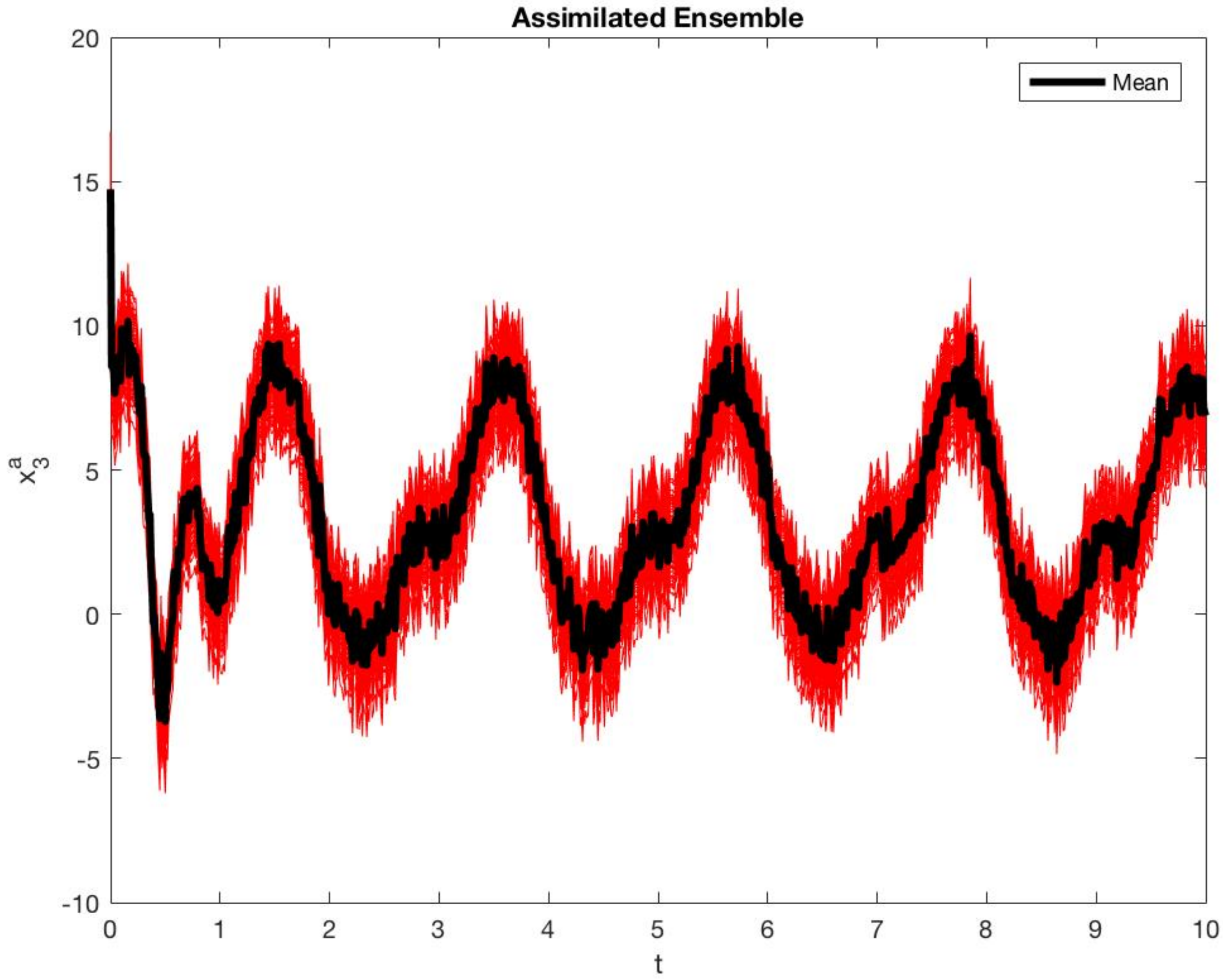
METHODS

- **Create an ensemble of x^b's**



METHODS

- **Assimilate**



PARAMETERIZATION ANALYSIS OF LORENZ '96

- **F in Lorenz '96 represented by function:[4]**

$$F_i = f_0 + \theta_{k-1} \sin(\frac{2\pi}{\theta_k} i)$$

- **k number of F members ➔ big uncertainty [4]**
- **Greater variance in means from running multiple times**

[4] *Two–Stage Filtering for Joint…* Santitissadeekorn

PARTICLE FILTER

- **Estimation of parameter state**

- **Assigns weights to particles based on distribution**

- **In our case ➔ Particle Filter determines which θ_k's are most common/effective in reducing variance [4]**

[4] *Two–Stage Filtering for Joint…* Santitissadeekorn

CONTINUED RESEARCH

- **Dual Filtering Algorithm [4]:**

  (At each assimilation cycle)

  - Parameterization (Particle Filter) ➔Ensemble Kalman Filter

- **Want to show effectiveness (low variance) from reducing the number of Particle Filter cycles**
  - Run the filter a few times to achieve similar/ better results

[4] *Two–Stage Filtering for Joint…* Santitissadeekorn

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REFERENCES

[1] *Data Assimilation: Part 1 Overview and Particle Filters*

Elaine Spiller

[2] *Ensemble-Based Atmospheric Assimilation: A Tutorial*

Thomas M. Hamill

[3] *Aggressive Shadowing of a Low Dimensional Model of Atmospheric Dynamics*

Ross M. Lieb-Lappen, Christopher M. Danforth

[4] *Two Stage Filtering for Joint State Parameter Estimation*

Naratip Santitissadeekorn, Christopher Jones