Observing Patterns in 4-Cycle Graphs

Carissa Babcock

August 2, 2017

2017 Summer Marquette MSCS

REU Participant

Mentor: Dr. Kim Factor

Previous Work

- In previous work, Factor, Merz, and Sano did research on competition numbers
- It was discovered that 4-cycle (C_4) graphs have competition numbers that are greater than their (1,2)-step competition numbers
- Proposed the question if there are other graphs that hold this same pattern

My Research

- Examined patterns in $C_4 \cup C_4$
- Examined patterns of C_4 with pendants

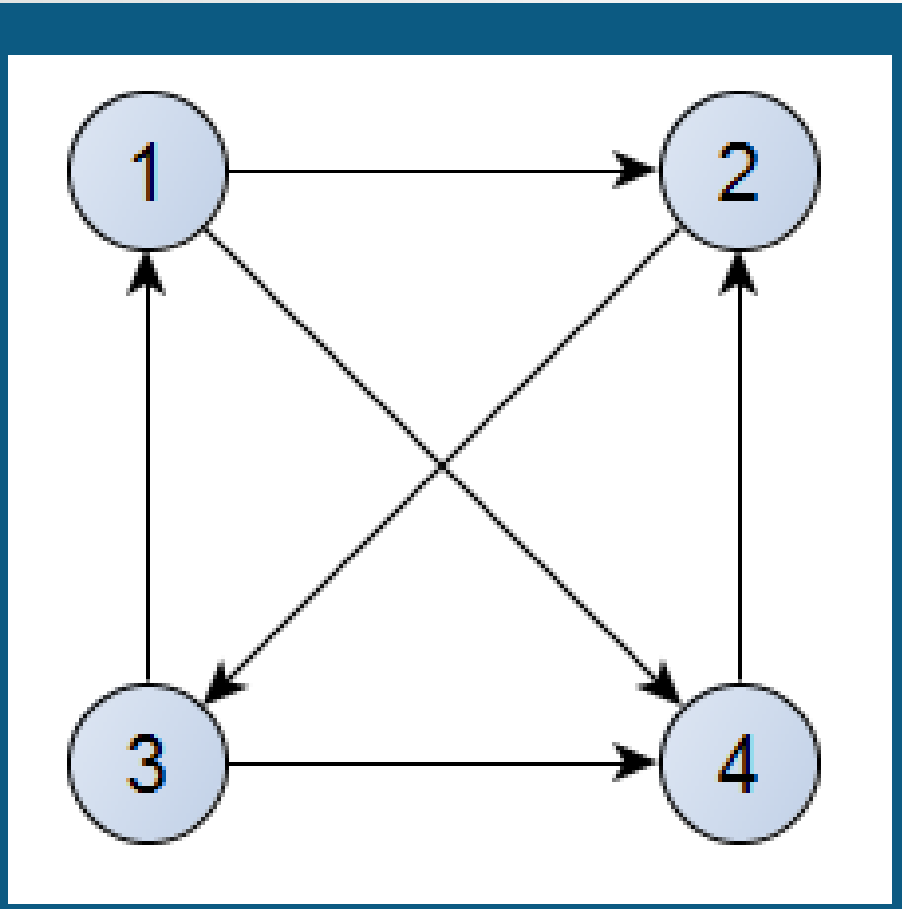
Importance of this Research

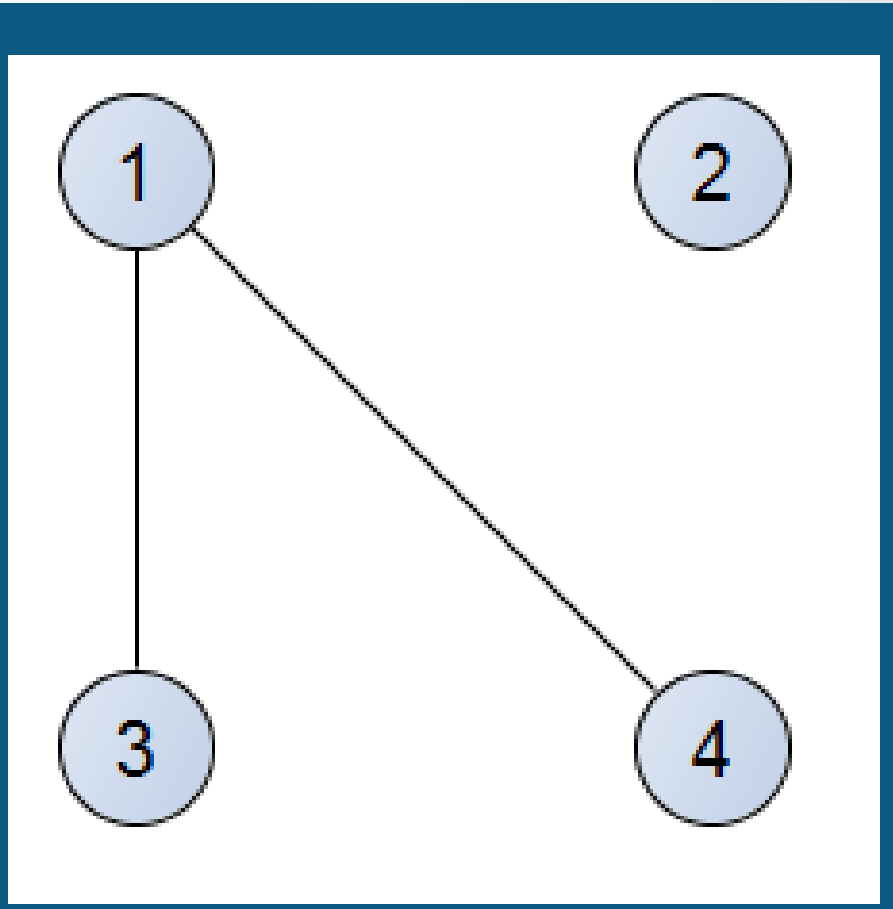
- Useful in Biology: can be used in ecological studies (food webs); can support Biology notions
- Useful in Mathematics: answers previous research question

Competition Number

- A competition number (denoted $\gamma(D)$) is the smallest nonnegative integer k in a graph G in which k isolated vertices and G is the competition graph of some acyclic digraph.

Competition Graph

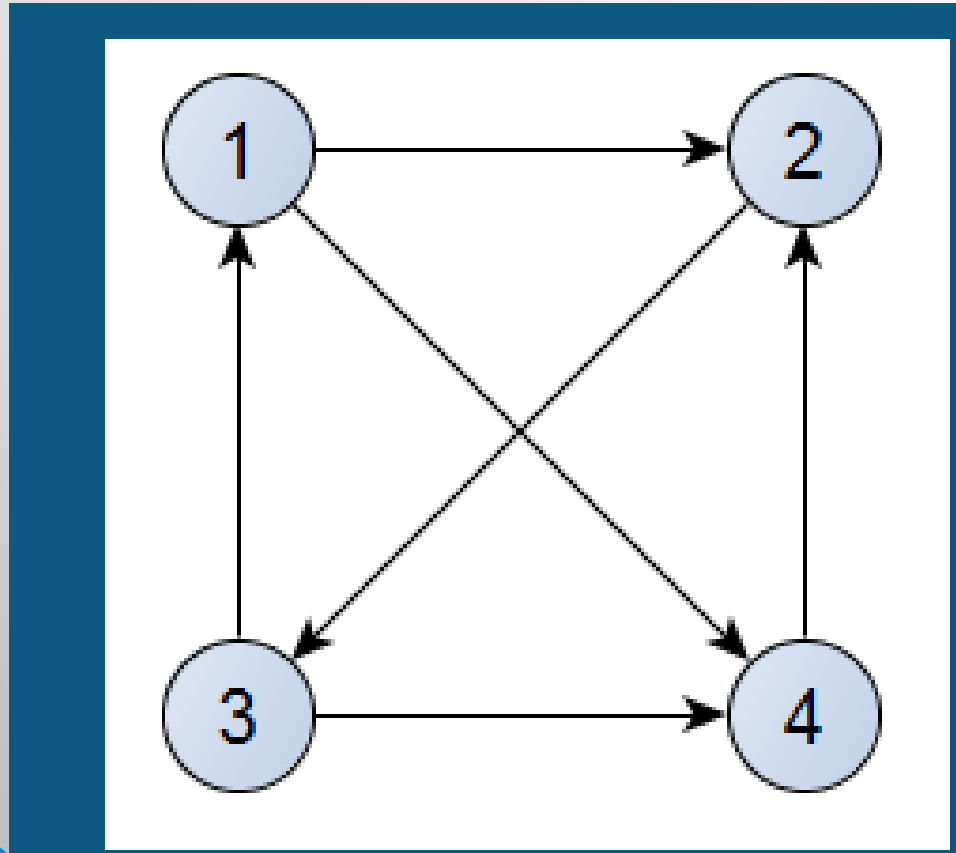


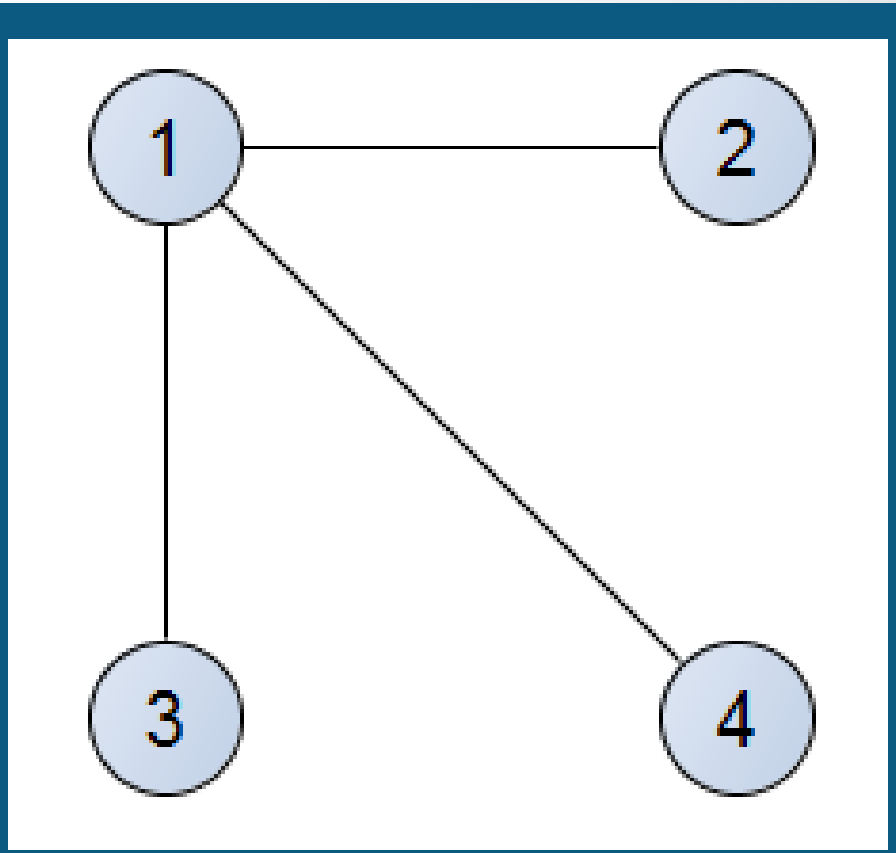


(1,2)-Step Competition Number

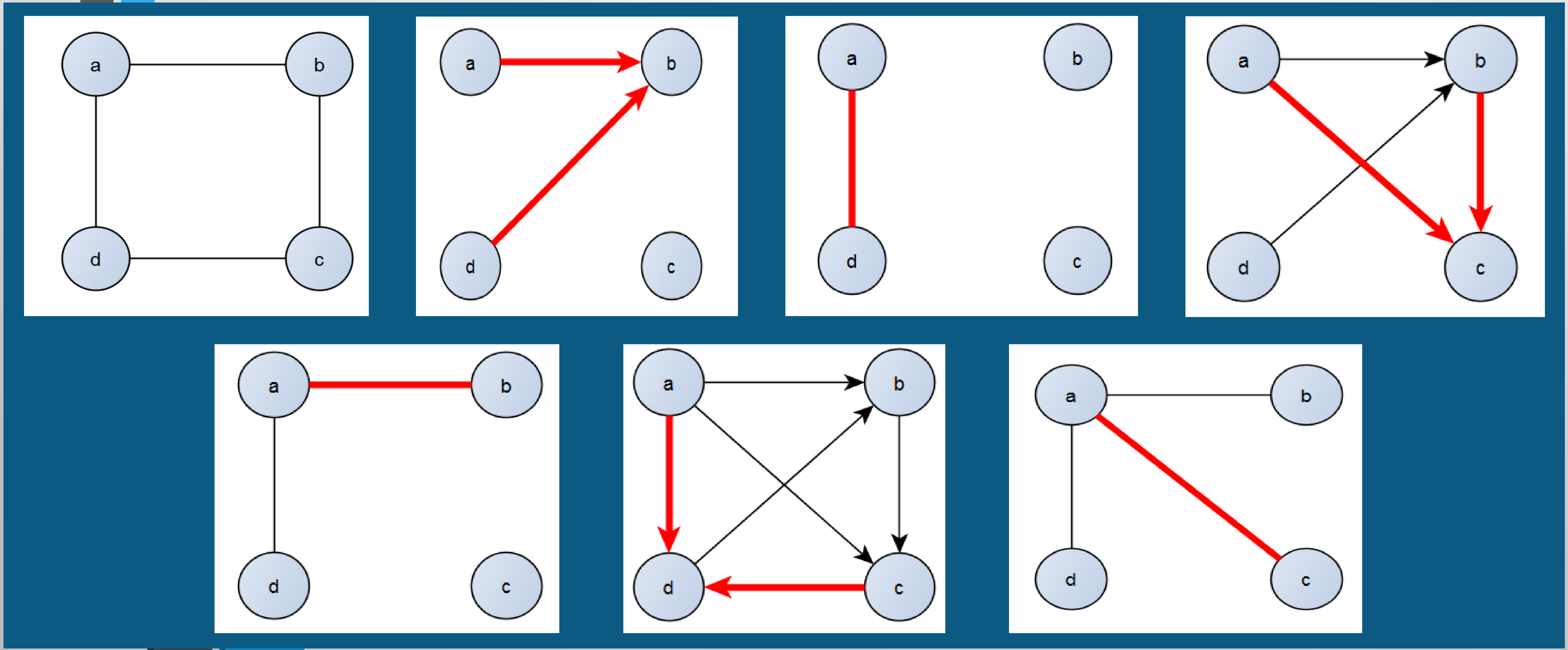
- A (1,2)-step competition number (denoted $\gamma_{(1,2)}(D)$) is the smallest nonnegative integer *k* in a graph *G* in which *k* isolated vertices and *G* is the (1,2)-step competition graph of some acyclic digraph.

(1,2)-Step Competition Graph

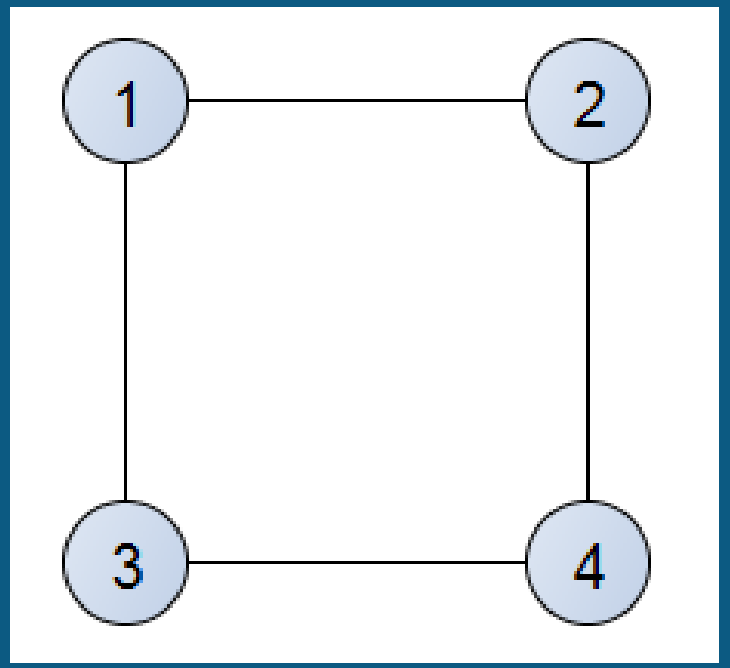


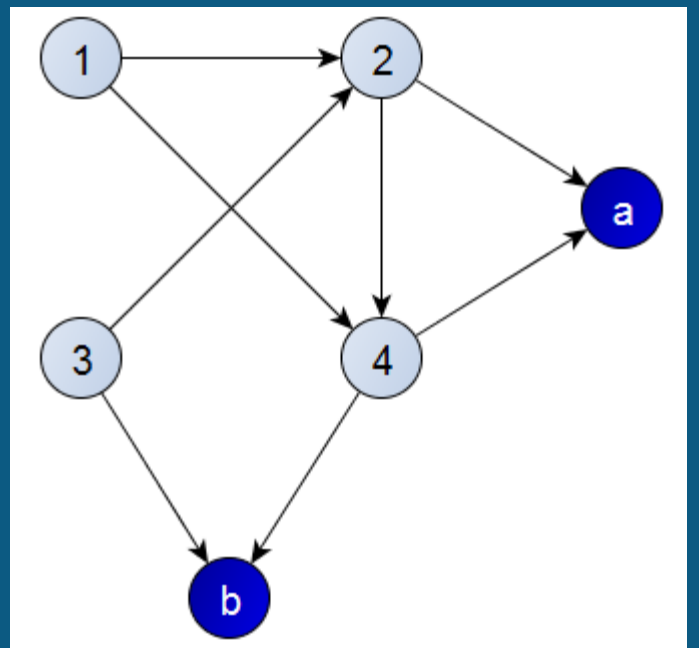


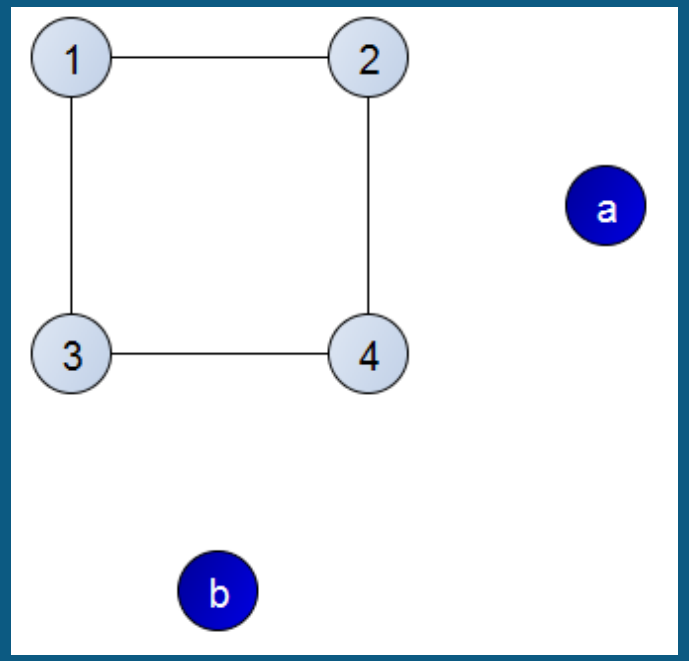
The Need for a Competition Number



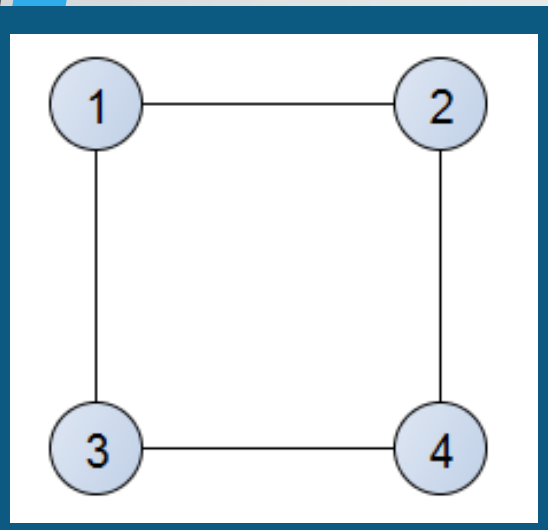
C_4 Competition Number

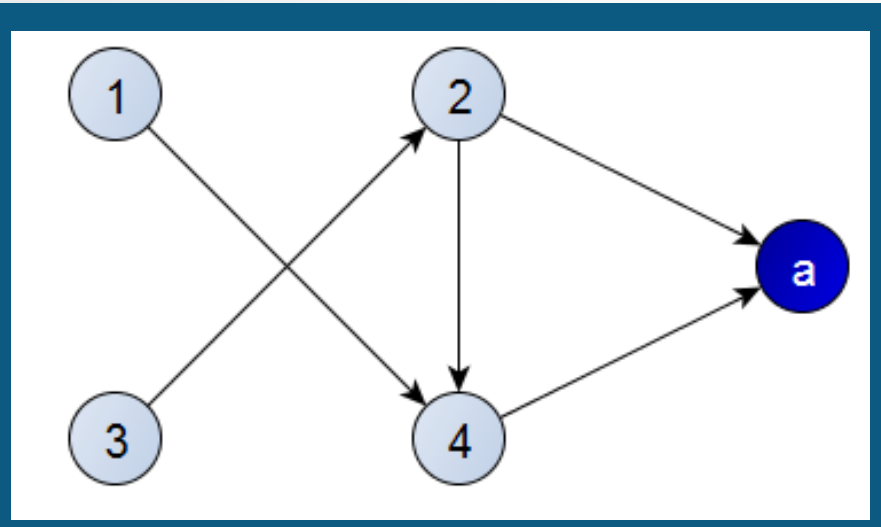


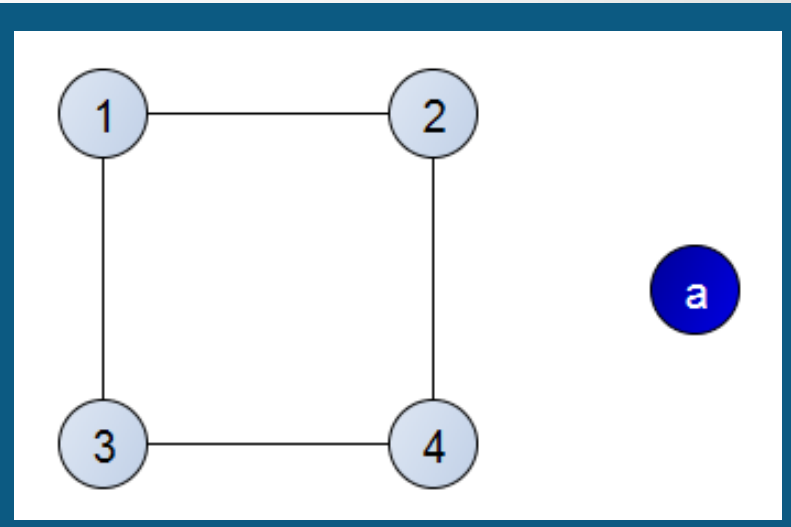




C_4 (1,2)-Step Competition Number

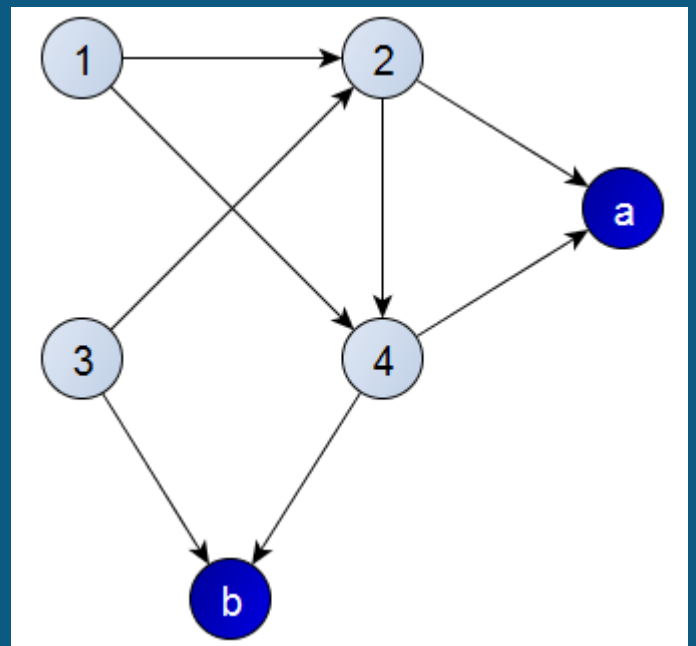




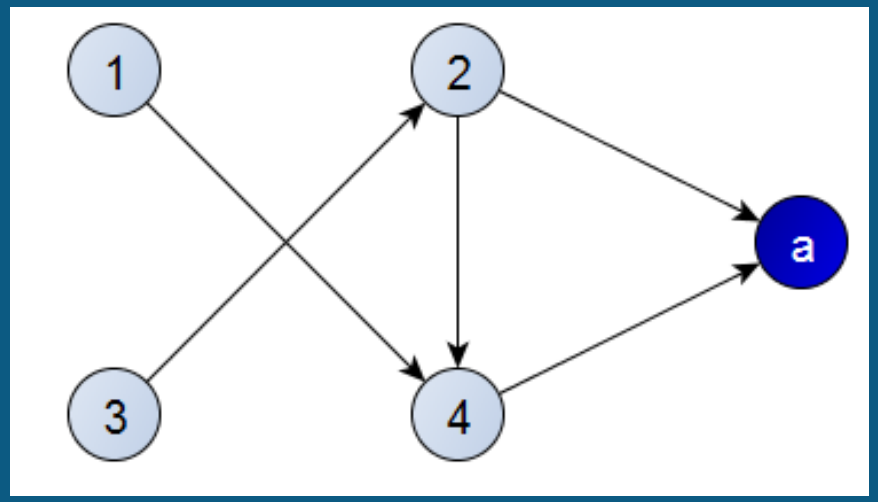


What Can we Study?

Competition Number



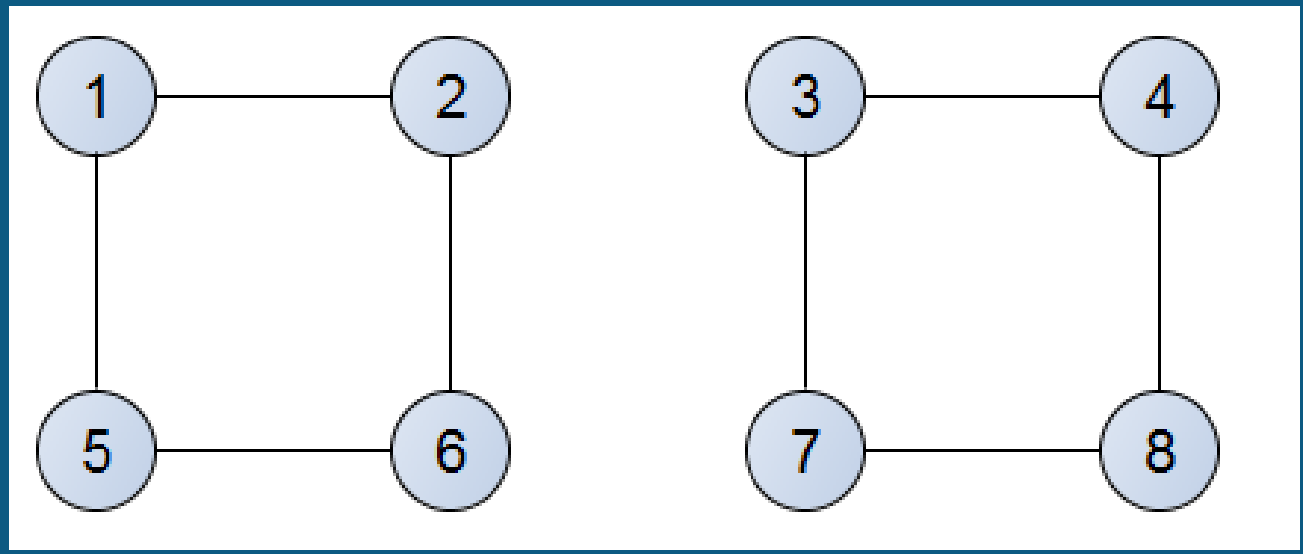
(1,2)-step Competition Number



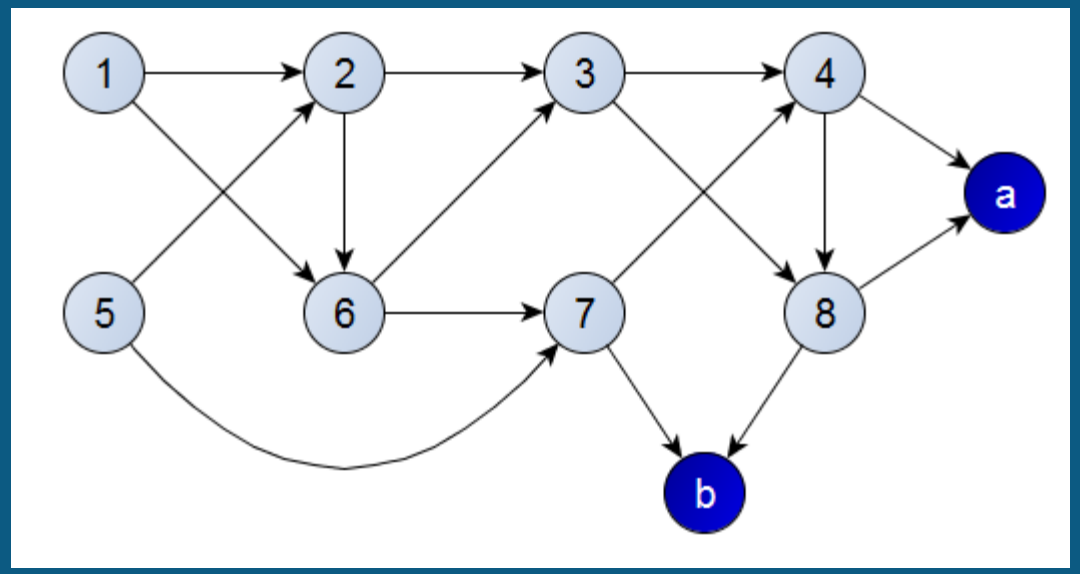
Question to Consider

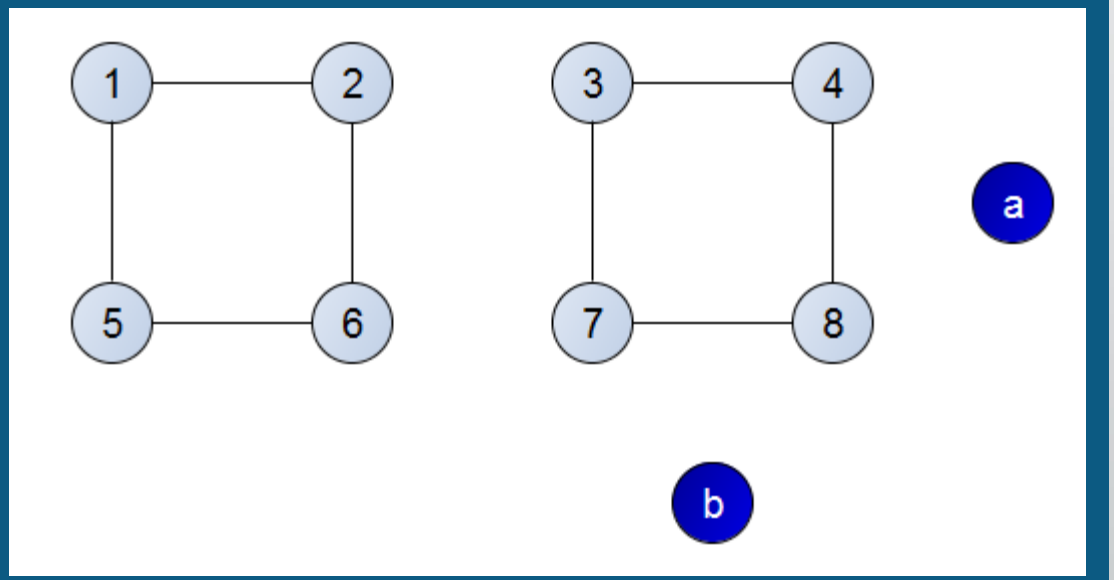
What patterns can be observed from looking at C_4 to see if there are other graphs in which $\gamma(D) > \gamma_{(1,2)}(D)$?

$C_4 \cup C_4$

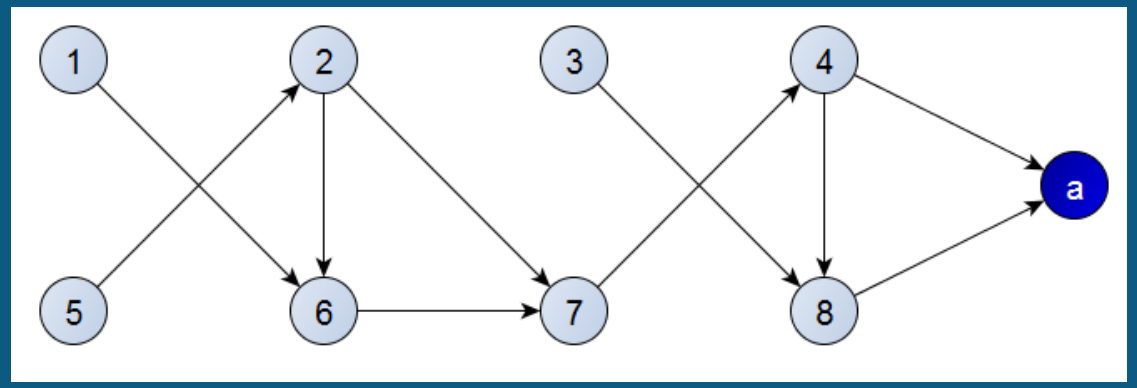


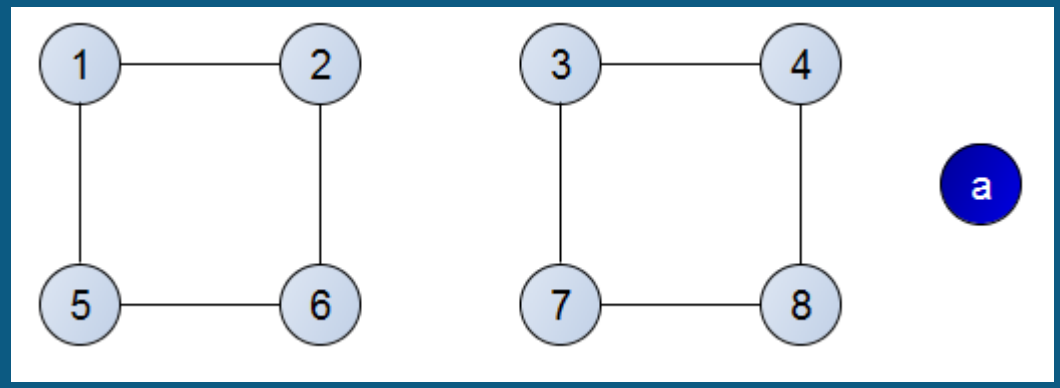
$C_4 \cup C_4$ Competition Number





$C_4 \cup C_4$ (1,2)-Step Competition Number





$C_4 \cup C_4$ Results

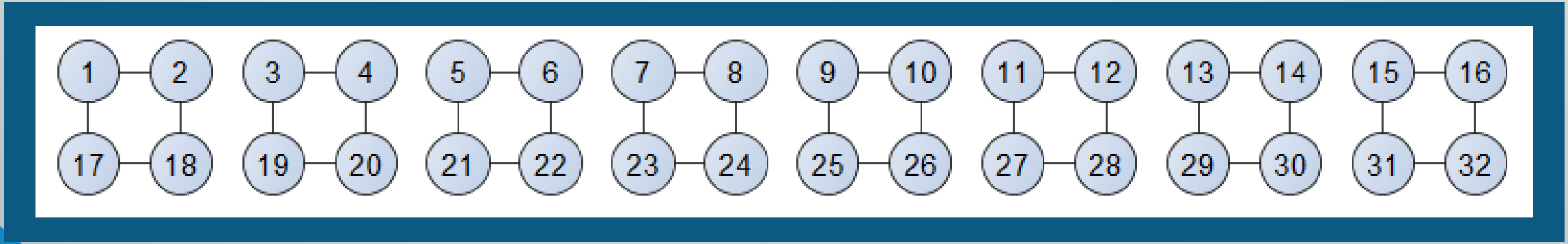
- It can be seen that:
  - $\gamma(C_4 \cup C_4) = 2$
  - $\gamma_{(1,2)}(C_4 \cup C_4) = 1$

Further Extensions of $C_4 \cup C_4$

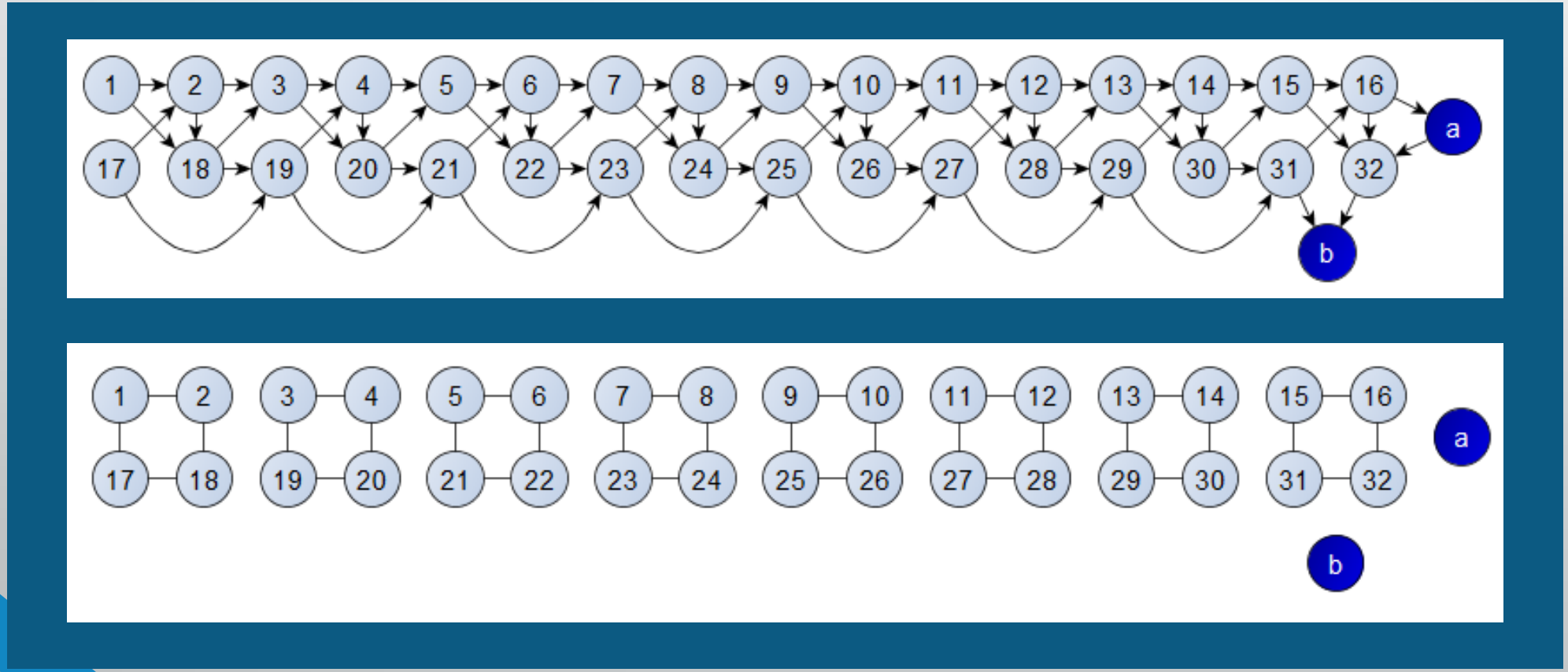
- The pattern in which $\gamma(G) = 2$ and $\gamma_{(1,2)}(G) = 1$ continues for multiple unions of C_4

Further Extensions of $C_4 \cup C_4$

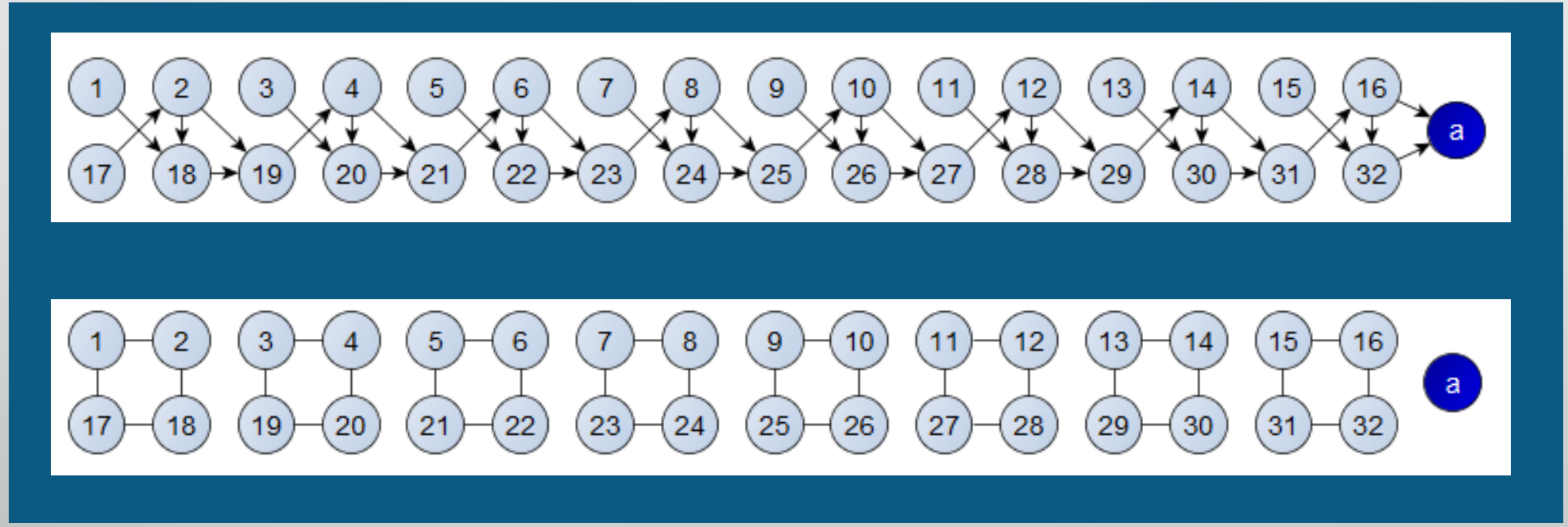
- 8 unions of C_4:



Further Extensions of $C_4 \cup C_4$: Competition Number



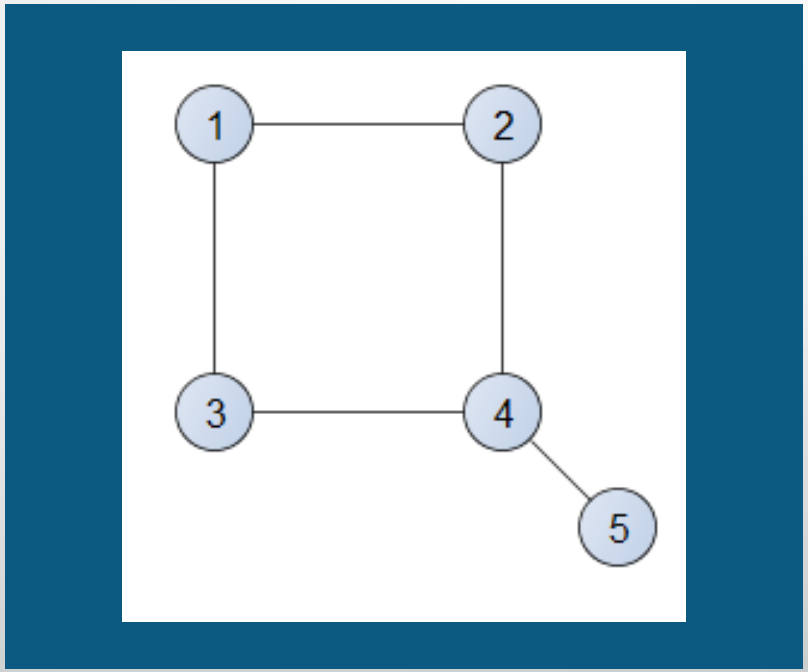
Further Extensions of $C_4 \cup C_4$: (1,2)-Step Competition Number



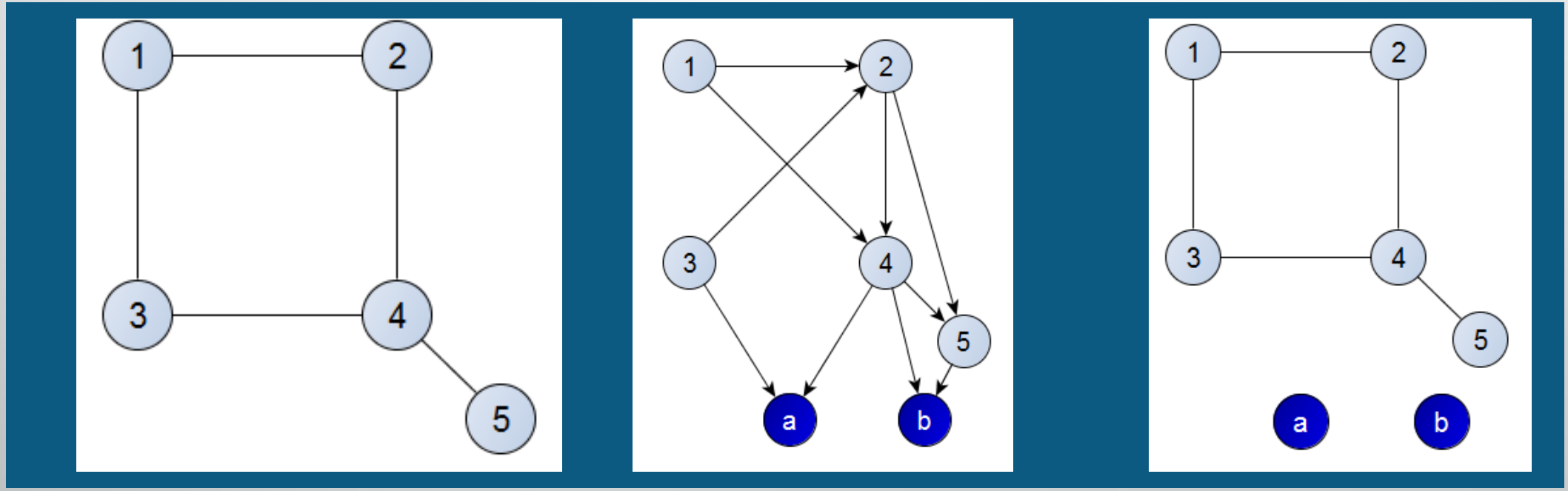
Further Extensions of $C_4 \cup C_4$: Results

- Graph *G* is eight unions of C_4:
  - $\gamma(G) = 2$
  - $\gamma_{(1,2)}(G) = 1$
- For *k* unions of C_4, $\gamma(D) > \gamma_{(1,2)}(D)$

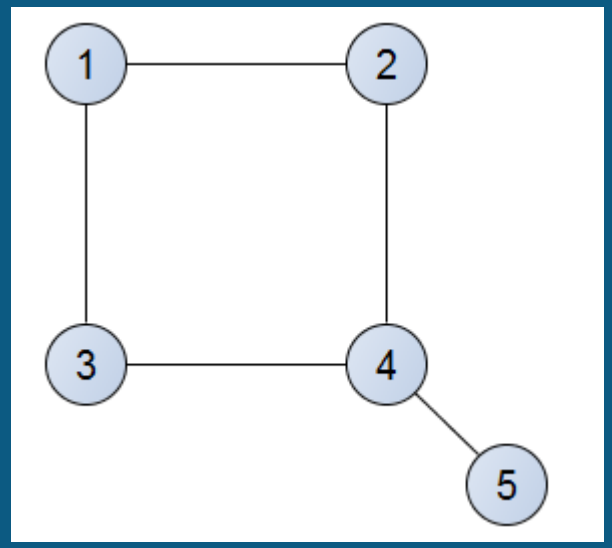
C_4 With Pendant

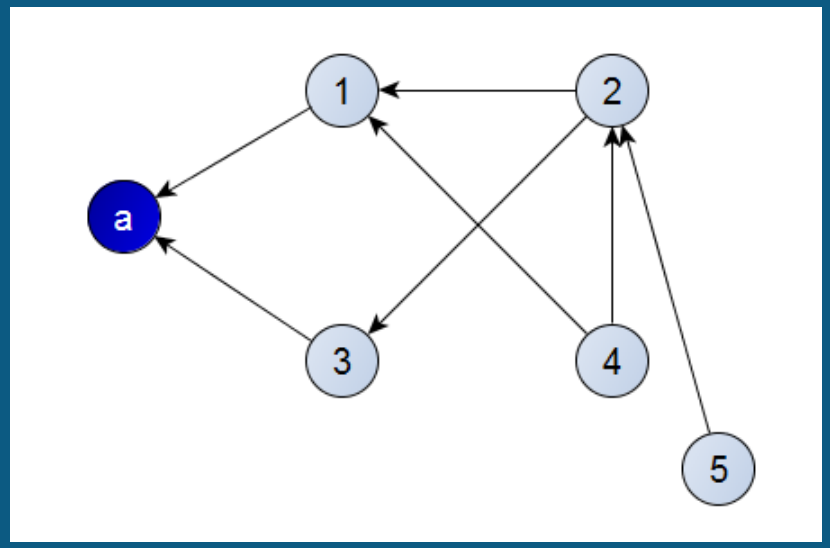


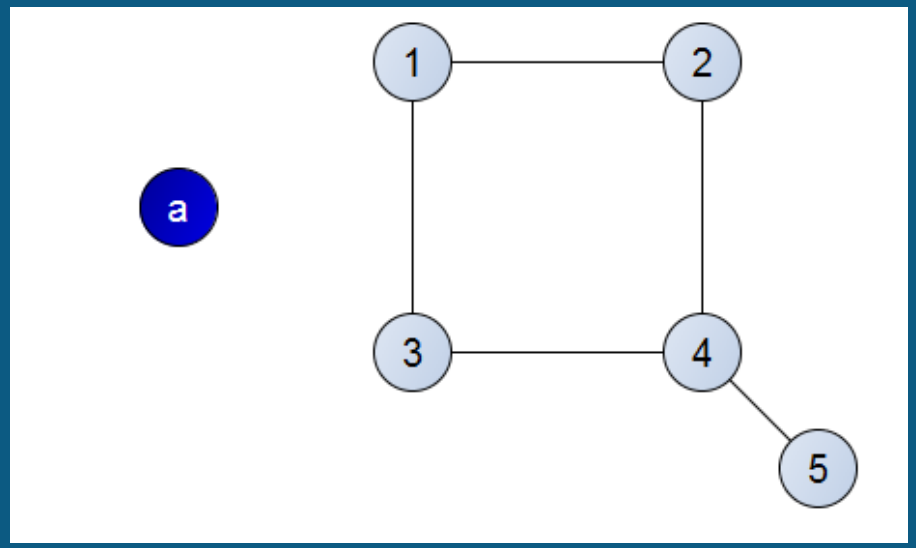
C_4 With Pendant Competition Number



C_4 With Pendant (1,2)-Step Competition Number





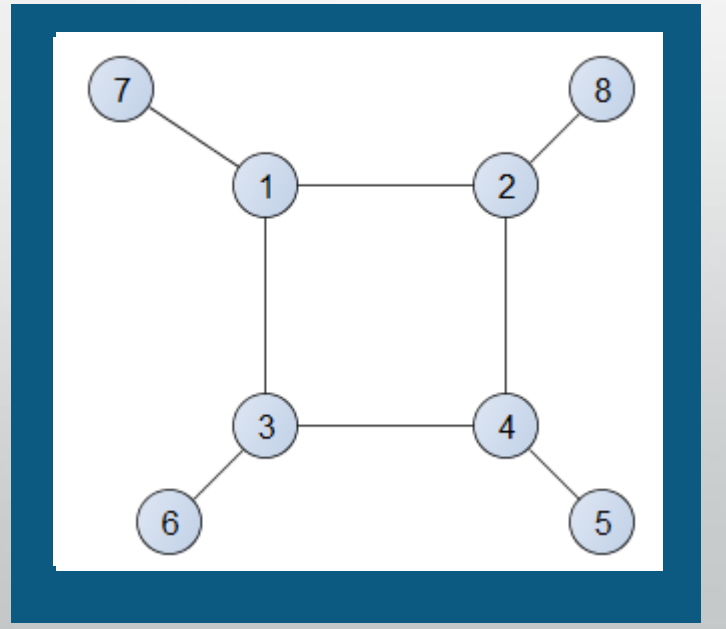


C_4 With Pendant Results

- G is a C_4 graph with one pendant:
  - $\gamma(G) = 2$
  - $\gamma_{(1,2)}(G) = 1$

C_4 With Pendants Future Work

- Currently working on adding additional pendants and seeing if there is any pattern in $\gamma(D)$ and $\gamma_{(1,2)}(D)$



Future Work

- Are there other extensions of C_4 graphs where $\gamma(D) > \gamma_{(1,2)}(D)$?
- Are there other basic graph structures besides C_4 graphs where this pattern remains true?
- Is there still a pattern when further extensions of pendants are used?
- What patterns exist when taking unions of C_4 graphs and adding an edge between them?

Acknowledgements

- National Science Foundation
- Marquette University MSCS
- Dr. Kim Factor
- Max Black

References

- Factor, K., Merz, S. *The (1,2)-step competition graph of a tournament.* Discrete Applies Mathematics. Volume 159, Issues 2-3
- Factor, K., Merz, S., Sano, Y. *The (1,2)-step competition number of a graph*. Congressus Numerantium 215, (2013) 153-161